

5-10 Differentiation - Chain Rule

Differentiate each function with respect to x .

1) $y = (x^3 + 3)^5$

$$\frac{dy}{dx} = 5(x^3 + 3)^4 \cdot 3x^2$$

$$= \boxed{15x^2(x^3 + 3)^4}$$

2) $y = (-3x^5 + 1)^3$

$$\frac{dy}{dx} = 3(-3x^5 + 1)^2 \cdot -15x^4$$

$$= \boxed{-45x^4(-3x^5 + 1)^2}$$

3) $y = (-5x^3 - 3)^3$

$$\frac{dy}{dx} = 3(-5x^3 - 3)^2 \cdot -15x^2$$

$$= \boxed{-45x^2(-5x^3 - 3)^2}$$

4) $y = (5x^2 + 3)^4$

$$\frac{dy}{dx} = 4(5x^2 + 3)^3 \cdot 10x$$

$$= \boxed{40x(5x^2 + 3)^3}$$

5) $f(x) = \sqrt[4]{-3x^4 - 2} = (-3x^4 - 2)^{\frac{1}{4}}$

$$f'(x) = \frac{1}{4}(-3x^4 - 2)^{-\frac{3}{4}} \cdot -12x^3$$

$$= \boxed{-\frac{3x^3}{(-3x^4 - 2)^{\frac{3}{4}}}}$$

6) $f(x) = \sqrt{-2x^2 + 1} = (-2x^2 + 1)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(-2x^2 + 1)^{-\frac{1}{2}} \cdot -4x$$

$$= \boxed{-\frac{2x}{(-2x^2 + 1)^{\frac{1}{2}}}}$$

7) $f(x) = \sqrt[3]{-2x^4 + 5} = (-2x^4 + 5)^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3}(-2x^4 + 5)^{-\frac{2}{3}} \cdot -8x^3$$

$$= \boxed{-\frac{8x^3}{3(-2x^4 + 5)^{\frac{2}{3}}}}$$

8) $y = (-x^4 - 3)^{-2}$

$$\frac{dy}{dx} = -2(-x^4 - 3)^{-3} \cdot -4x^3$$

$$= \boxed{\frac{8x^3}{(-x^4 - 3)^3}}$$

These problems require multiple rules!!

9) $y = (3x^3 + 1)(-4x^2 - 3)^4$

$$\begin{aligned}\frac{dy}{dx} &= (3x^3 + 1) \cdot 4(-4x^2 - 3)^3 \cdot -8x + (-4x^2 - 3)^4 \cdot 9x^2 \\ &= x(-4x^2 - 3)^3(-132x^3 - 32 - 27x)\end{aligned}$$

10) $y = \frac{(x^3 + 4)^5}{3x^4 - 2}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(3x^4 - 2) \cdot 5(x^3 + 4)^4 \cdot 3x^2 - (x^3 + 4)^5 \cdot 12x^3}{(3x^4 - 2)^2} \\ &= \frac{3x^2(x^3 + 4)^4(11x^4 - 10 - 16x)}{(3x^4 - 2)^2}\end{aligned}$$

11) $y = ((x + 5)^5 - 1)^4$

$$\begin{aligned}\frac{dy}{dx} &= 4((x + 5)^5 - 1)^3 \cdot 5(x + 5)^4 \\ &= 20((x + 5)^5 - 1)^3 \cdot (x + 5)^4\end{aligned}$$

12) $y = (5x^3 - 3)^5 \sqrt[4]{-4x^5 - 3}$

$$\begin{aligned}\frac{dy}{dx} &= (5x^3 - 3)^5 \cdot \frac{1}{4}(-4x^5 - 3)^{-\frac{3}{4}} \cdot -20x^4 + (-4x^5 - 3)^{\frac{1}{4}} \cdot 5(5x^3 - 3)^4 \cdot 15x^2 \\ &= \frac{5x^2(5x^3 - 3)^4(-65x^5 - 45 + 3x^2)}{(-4x^5 - 3)^{\frac{3}{4}}}\end{aligned}$$

Critical thinking question:

13) Give a function that requires three applications of the chain rule to differentiate. Then differentiate the function.

Many answers; Ex $y = (((2x + 1)^5 + 2)^6 + 3)^7 \rightarrow \text{Whoa!!}$

$$\frac{dy}{dx} = 7(((2x + 1)^5 + 2)^6 + 3)^6 \cdot 6((2x + 1)^5 + 2)^5 \cdot 5(2x + 1)^4 \cdot 2$$